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Kinetic behaviours of aggregate growth driven by time-dependent migration, birth and death

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Abstract

We propose a dynamic growth model to mimic some social phenomena, such as the evolution of cities' population, in which monomer migrations occur between any two aggregates and monomer birth/death can simultaneously occur in each aggregate. Considering the fact that the rate kernels of migration, birth and death processes may change with time, we assume that the migration rate kernel is $ijf(t)$, and the self-birth and death rate kernels are $ig_1(t)$ and $ig_2(t)$, respectively. Based on the mean-field rate equation, we obtain the exact solution of this model and then discuss semi-quantitatively the scaling behaviour of the aggregate size distribution at large times. The results show that in the long-time limit, (i) if $\int_0^t g_1(t')\,\mathrm{d}t' \big/ \int_0^t g_2(t')\,\mathrm{d}t' \geqslant 1$ or $\exp\left\{\int_0^t [g_2(t') - g_1(t')]\,\mathrm{d}t'\right\} \big/ \int_0^t f(t')\,\mathrm{d}t' \to 0$, the aggregate size distribution $a_k(t)$ can obey a generalized scaling form; (ii) if $\int_0^t g_1(t')\,\mathrm{d}t' \big/ \int_0^t g_2(t')\,\mathrm{d}t' \to 0$ and $\exp \int_0^t [g_2(t') - g_1(t')\,\mathrm{d}t' \big/ \int_0^t f(t')\,\mathrm{d}t' \to \infty$, $a_k(t)$ can take a scale-free form and decay exponentially in size k; (iii) $a_k(t)$ will satisfy a modified scaling law in the remaining cases. Moreover, the total mass of aggregates depends strongly on the net birth rate $g_1(t) - g_2(t)$ and evolves exponentially as $\exp\left\{\int_0^t [g_1(t') - g_2(t')]\,\mathrm{d}t'\right\}$, which is in qualitative agreement with the evolution of the total population of a country in real world.

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1. Introduction

Aggregate growth is a common and important phenomenon in many fields of nature and social science, such as physics, chemistry, biology and demography [1–4]. In the past few decades, many investigations have been focused on the kinetics of various aggregation processes. The general mechanisms arising in diverse fields of nature and social science include binary coalescence, migration, fragmentation, annihilation and so on [5–10]. Recently, much attention has been devoted to aggregate growth driven by migration or exchange. For example, Ispolatov, Krapivsky, and Redner introduced the migration mechanism to solve the problem of wealth distribution in economical interactions [11], and Leyvraz and Redner proposed a migration-driven aggregate growth model to mimic the evolution of city population [12]. In these models, aggregate growth takes place through the biased migration mechanism, $A_k + A_l \xrightarrow{K(k,l)} A_{k-1} + A_{l+1}$ $(k \leqslant l)$, where A_i denotes an aggregate (corresponding, a city or agent) consisting of i monomers (e.g., individuals). That is, monomers prefer to transfer from the small aggregate A_k to the large aggregate A_l at the rate $K(k, l)$. In recent decades, the migration mechanism is extensively used to discover the kinetic behaviour of aggregate growth. Ke and Lin further investigated the kinetics of a general unbiased migration-driven aggregation system to show how the rate kernel $K(k, l) \propto kl^{\mu}$ influences the aggregate size distribution (μ is the migration rate kernel index, which reflects the activity of the aggregates in emigration and immigration) [13]. The results showed that the aggregate size distribution can approach different scaling forms for different values of μ. Moreover, Ben–Naim and Krapivsky made a general study of exchange-driven growth with a generalized homogeneous rate kernel [14]. These works exhibited that the migration- or exchange-driven aggregation processes have much abundant kinetic behaviours.

Actually, we realize that monomer birth and death also play important roles in aggregate growth processes (e.g., the evolution of city population). For example, an aggregate (e.g., city) consisting of i individuals may grow into a large aggregate consisting of $i + 1$ individuals through monomer birth. Similarly, an aggregate can also be reduced to a small one through monomer death processes. Such mechanisms can be described respectively by the schemes, $A_i \xrightarrow{J_1(i)} A_{i+1}$ and $A_i \xrightarrow{J_2(i)} A_{i-1}$, where $J_1(i)$ and $J_2(i)$ are the self-birth and self-death rate kernels which depend only on the size of the mother aggregate. Lin and Ke proposed a migration-driven growth model with monomer birth and death to mimic the evolution of city population and individual wealth more naturally, and they emphatically investigated how the self-birth and self-death rate kernels decided the form which the aggregate size distribution takes [15, 16]. The results showed that migration, self-birth and self-death processes are very common in nature and should be introduced in some aggregate growth systems.

It should be pointed out that in the above-mentioned models all the rate kernels are assumed to be independent of time. Moreover, in the literature, most of the research works on aggregate growth have only paid attention to those time-independent rate kernels. However, in some situations, the rate kernels of aggregation processes may have relation with time. For example, in chemical reaction processes, the occurrence that two reactants coagulate into a big one is usually dependent on the catalytic ability of the catalyst. And the catalytic ability of the catalyst may not keep invariant for some factors such as ageing, which may cause the coagulability of reactants decay or increase with time. Moreover, it is well known that in a real society the migration and self-birth, and self-death processes of population may be dependent on time actually. For instance, the unbalanced economic development among cities, which varies continuously with time, can lead to a large-scale population migration (e.g., more and more people migrate from a poor city to a rich one). Similarly, we realize that when the city's economy depresses, the migration rate of population may slow down. On the other hand, the self-birth and self-death processes also influence the evolution of city population. And the rates of self-birth and self-death can change with some factors, such as fertility concept, the related policy of local government and the social medical care system. In other words, the rate kernels of self-birth and self-death are both time dependent. Ke and his co-workers investigated the reversible aggregation process with time-dependent rate kernels and obtained the asymptotic solution of the cluster size distribution [17]. Straube and Falcke studied cluster–cluster aggregation and fragmentation with a periodically modulated binding rate, and their result is relevant whenever clustering can be externally controlled [18]. These investigations showed that the kinetic behaviour of aggregate growth is also dependent strongly on the concrete forms of the time-dependent rate kernels.

According to the above-mentioned reasons, we further investigate the migration model with birth and death, in which all the reaction rate kernels are time dependent. The unbiased migration process can be described as the reaction $A_i + A_j \xrightarrow{I(i,j;t)} A_{i-1} + A_{j+1}$. Here, $I(i, j; t)$ is the time-dependent migration rate kernel at which one monomer migrates from the aggregate A_i to another aggregate A_j at time t. The processes of birth and death can be described by the monomer reactions [15], $A_i \xrightarrow{J_1(i;t)} A_{i+1}$ and $A_i \xrightarrow{J_2(i;t)} A_{i-1}$, where $J_1(i; t)$ is the monomer birth rate kernel and $J_2(i; t)$ the death rate kernel at time t. We believe that such an aggregate growth model with time-dependent rate kernels can mimic some phenomena in biology and social science, such as the evolution of city population and animal group, more accurately. Moreover, it is also of theoretical interest to study the dependence of the kinetics on time-dependent rate kernels.

The rest of this paper is organized as follows. In section 2, we investigate the analytic solution of the model by means of the rate equation approach and then discuss the evolution properties of the total mass, the total number and the aggregate size distribution. Finally, a brief summary is given in section 3.

2. Analytic solution of reversible migration processes with birth and death

In this paper, the theoretical approach to the kinetics of reversible migration processes with birth and death is based on the mean-field theory, which assumes that fluctuations in densities of reactants are ignored and all aggregates are considered to be homogeneously distributed in place throughout the processes. In the mean-field limit, we can use the Smoluchowski rate equation to investigate the analytic solution of such a model. Let $a_k(t)$ be the concentration of the aggregate A_k at time t. Based on [15, 16], we write the governing rate equation for our system as follows:

$$\frac{\mathrm{d}a_k}{\mathrm{d}t} = \sum_{j=1}^{\infty} I(k+1, j; t)a_{k+1}a_j + \sum_{j=1}^{\infty} I(j, k-1; t)a_j a_{k-1} - \sum_{j=1}^{\infty} [I(k, j; t) + I(j, k; t)]a_k a_j + J_1(k-1; t)a_{k-1} - J_1(k; t)a_k + J_2(k+1; t)a_{k+1} - J_2(k; t)a_k. \tag{1}$$

In equation (1), the first two terms account for the gain in $a_k(t)$ due to the migrations $A_{k+1} + A_j \to A_k + A_{j+1}$ and $A_j + A_{k-1} \to A_{j-1} + A_k$ ($j = 1, 2, \ldots$). The third and fourth terms account for the loss in $a_k(t)$ due to the migration $A_k + A_j \to A_{k-1} + A_{j+1}$ and its equiprobable process $A_j + A_k \to A_{j-1} + A_{k+1}$ ($j = 1, 2, \ldots$). The fifth and sixth terms represent the gain and loss in $a_k(t)$ caused by the self-birth processes $A_{k-1} \to A_k$ (gain) and $A_k \to A_{k+1}$ (loss), respectively. The last two terms represent the gain and loss in $a_k(t)$ due to the self-death processes $A_{k+1} \to A_k$ (gain) and $A_k \to A_{k-1}$ (loss), respectively.

In general, the rate kernels of migration, birth and death processes are not only related to time but also dependent on the sizes of reactant aggregates. However, similar to the rate equations for binary coagulation models [6–10], the governing rate equation (1) in this model is also an infinite set of nonlinear differential equations. Thus, it is difficult to solve analytically equation (1) in general cases with general rate kernels. Some useful mathematical techniques, such as the application of scaling ansatz or Laplace transforms, have been employed to solve such nonlinear rate equations (see, e.g., [8, 10–12]). In our previous works [19], we used the method of separation of variables to obtain the analytic solution of the rate equation for an aggregation–fragmentation model with special rate kernels. In order to obtain the analytic solution of equation (1) and then discuss the evolution properties of the aggregate size distribution, we also focus on a simple case with special rate kernels in this work. We assume that all monomers, individually or among aggregates, have the same characteristics including physical properties and reaction activities. Thus, the migration rate kernel is symmetrical and directly proportional to the size of the emigrating aggregate and that of the immigrating aggregate, namely, $I(i, j; t) = ijf(t)$, where the time-dependent function $f(t)$ describes the probability of migration per monomer at time t. Meanwhile, both the self-birth and self-death rate kernels are also time dependent and directly proportional to the size of the mother aggregate, i.e., $J_1(i; t) = ig_1(t)$ and $J_2(i; t) = ig_2(t)$. Here, $g_1(t)$ denotes the probability of a monomer producing a new one and $g_2(t)$ denotes the death probability of an old monomer at time t. These rate kernels may be sound for identical particle systems and social systems such as the population distribution of cities.

With these hypotheses equation (1) can be rewritten as

$$\frac{\mathrm{d}a_k}{\mathrm{d}t} = [(k+1)a_{k+1} + (k-1)a_{k-1} - 2ka_k]M_1 f + [(k-1)a_{k-1} - ka_k]g_1 + [(k+1)a_{k+1} - ka_k]g_2 \quad (2)$$

with the shorthand notation $M_1(t) = \sum_{j=1}^{\infty} ja_j(t)$. Obviously, $M_1(t)$ denotes the total mass of the species at time t.

In this work, we consider a simple but important case in which there only exist monomer aggregates at $t = 0$ and the initial concentration is equal to A_0. Then the initial condition is

$$a_k(0) = A_0\,\delta_{k1}. \quad (3)$$

Under the monodisperse initial condition, we can solve equation (2) with the help of the ansatz (see, e.g., [19, 20])

$$a_k(t) = A(t)\,[a(t)]^{k-1}. \quad (4)$$

Substituting the ansatz (4) into equation (2), we can derive the following differential equations:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = (1-a)^2 M_1 f + (1-a)g_1 + (a^2 - a)g_2 \quad (5)$$

$$\frac{\mathrm{d}A}{\mathrm{d}t} = 2A(a-1)M_1 f - Ag_1 + A(2a-1)g_2 \quad (6)$$

with the corresponding initial condition

$$a(0) = 0 \quad A(0) = A_0. \quad (7)$$

Then the problem reduces to determining the solutions of $a(t)$ and $A(t)$. It should be pointed out that the mathematical technique employed above is a sort of the method of separation of variables only under the condition $0 < a(t) < 1$ for all t.

From equations (5) and (6) we can deduce

$$\frac{\mathrm{d}\ln M_1}{\mathrm{d}t} = \frac{2}{1-a}\frac{\mathrm{d}a}{\mathrm{d}t} + \frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}t} = g_1 - g_2. \quad (8)$$

The exact expression of the total mass of aggregates can then be derived,

$$M_1(t) = A_0 \exp\left[\int_0^t [g_1(t') - g_2(t')]\,\mathrm{d}t'\right]. \quad (9)$$

Substituting equation (9) into equation (5), we can easily deduce the following equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{1-a}\right)+[g_2(t)-g_1(t)]\frac{1}{1-a}=g_2(t)+A_0f(t)\exp\left[\int_0^t[g_1(t')-g_2(t')]\,\mathrm{d}t'\right]. \tag{10}$$

Equation (10) is a linear equation, which can be straightforwardly solved to yield

$$a(t)=1-\frac{\exp\{[\bar{g}_2(t)-\bar{g}_1(t)]t\}}{\int_0^t g_2(t')\exp\{[\bar{g}_2(t')-\bar{g}_1(t')]t'\}\,\mathrm{d}t'+A_0\bar{f}(t)t+1} \tag{11}$$

where

$$\bar{f}(t)=\frac{1}{t}\int_0^t f(t')\,\mathrm{d}t' \qquad \bar{g}_1(t)=\frac{1}{t}\int_0^t g_1(t')\,\mathrm{d}t' \qquad \bar{g}_2(t)=\frac{1}{t}\int_0^t g_2(t')\,\mathrm{d}t'. \tag{12}$$

Here, $\bar{f}(t)$ denotes the average probability of migration per monomer in the time range of 0 to t, and $\bar{g}_1(t)$ and $\bar{g}_2(t)$ denote the average probability of monomer birth and that of monomer death, respectively. Correspondingly, in the time range of 0 to t, the average migration rate kernel is $\bar{I}(i,j;t)=\bar{f}(t)ij$, while the average birth and death rate kernels are $\bar{J}_1(i;t)=i\bar{g}_1(t)$ and $\bar{J}_2(i;t)=i\bar{g}_2(t)$, respectively.

By using the expression of the total mass, we readily deduce

$$A(t)=M_1(t)[1-a(t)]^2. \tag{13}$$

Thus we obtain the exact solution of the aggregate size distribution for arbitrary $f(t)$, $g_1(t)$ and $g_2(t)$,

$$a_k(t)=\frac{A_0\exp\{[\bar{g}_2(t)-\bar{g}_1(t)]t\}}{\left\{\int_0^t g_2(t')\exp\{[\bar{g}_2(t')-\bar{g}_1(t')]t'\}\,\mathrm{d}t'+A_0\bar{f}(t)t+1\right\}^2} \times\left\{1-\frac{\exp\{[\bar{g}_2(t)-\bar{g}_1(t)]t\}}{\int_0^t g_2(t')\exp\{[\bar{g}_2(t')-\bar{g}_1(t')]t'\}\,\mathrm{d}t'+A_0\bar{f}(t)t+1}\right\}^{k-1}. \tag{14}$$

It is also important to determine the total number of aggregates,

$$M_0(t)=\sum_{j=1}^{\infty}a_j(t)=\frac{A(t)}{1-a(t)} =\frac{A_0}{\int_0^t g_2(t')\exp\{[\bar{g}_2(t')-\bar{g}_1(t')]t'\}\,\mathrm{d}t'+A_0\bar{f}(t)t+1}. \tag{15}$$

When the time functions $f(t)$, $g_1(t)$ and $g_2(t)$ are given, one can readily understand the evolution behaviour of the aggregate size distribution by analysing equation (14). In the following subsections, we will semi-quantitatively discuss the scaling properties of the aggregate size distribution at large times. Here we focus on fairly real cases such as the evolution of city population. It is well known that the social economy can affect the social activities to a certain extent. For example, with fast economic development, the public transportation is getting more and more convenient, and accordingly, this situation makes population migration more frequent. We define $f(t)$ in the migration kernel is a positive and increasing function, or at least asymptotically so at large times, namely, $\dot{f}(t)\geqslant 0$ $(t\gg 1)$. Moreover, with the effect of people's fertility concept the self-birth rate may change with time, and recently it decreases with time especially in developed countries. On the other hand, with the help of the social medical care system the self-death rate may also decrease with time. Thus, $g_1(t)$ and $g_2(t)$ may be positive and decreasing functions. Assume that $\dot{g}_1(t)\leqslant 0$ and $\dot{g}_2(t)\leqslant 0$ at $t\gg 1$. We then investigate analytically the kinetic behaviour of such a relatively real system in the following cases.

2.1. The case with $\bar{g}_1(t)/\bar{g}_2(t) \to \infty$ at $t \to \infty$

Firstly, we investigate the $\lim_{t\to\infty} \bar{g}_1(t)/\bar{g}_2(t) \to \infty$ case. In the long-time limit, the value of $\bar{g}_2(t)$ can be ignored comparing to that of $\bar{g}_1(t)$. For this case, the total mass of the species increases with time and the species thus survives finally. Moreover, the total mass will diverge if the average birth rate decreases less rapidly than t^{-1}.

From equation (14) we deduce the asymptotic solution of $a_k(t)$ at large times,

$$a_k(t) \simeq A_0^{-1} t^{-2} [\bar{f}(t)]^{-2} \exp[-t\bar{g}_1(t)] \{1 - A_0^{-1} t^{-1} [\bar{f}(t)]^{-1} \exp[-t\bar{g}_1(t)]\}^{k-1}. \tag{16}$$

In the region of $t \gg 1$ and $k \gg 1$, equation (16) can further be rewritten as

$$a_k(t) \simeq A_0^{-1} t^{-2} [\bar{f}(t)]^{-2} \exp[-t\bar{g}_1(t)] \exp\left[-\frac{k}{A_0 t \bar{f}(t) \exp[t\bar{g}_1(t)]}\right]. \tag{17}$$

Obviously, equation (17) satisfies the generalized scaling form (see, e.g., [17, 19, 20])

$$a_k(t) \simeq [\alpha(t)]^{-1} \Phi\left[\frac{k}{S(t)}\right] \qquad S(t) \propto \beta(t) \tag{18}$$

where $\alpha(t)$ and $\beta(t)$ are increasing functions of time and $S(t)$ denotes the typical aggregate size of the system, which plays a role analogous to the correlation length in critical phenomena. For this case, the scaling function is exponential, namely, $\Phi(x) = \exp(-x)$, and the typical size $S(t)$ grows as $A_0 t \bar{f}(t) \exp[t\bar{g}_1(t)]$ in the long-time limit.

Moreover, from equation (15) we can obtain the asymptotic solution of $M_0(t)$ as follows:

$$M_0(t) \simeq t^{-1} [\bar{f}(t)]^{-1}. \tag{19}$$

Equation (19) indicates that the evolution behaviour of the total number depends crucially on the migration rate kernel. The total number decreases with time in the long-time limit and decays to zero finally. On the other hand, summing up equation (2), we can deduce $\dot{M}_0(t) = -[M_1(t)f(t) + g_2(t)]a_1(t)$. In this case, $M_1(t)f(t) \gg g_2(t)$ at $t \gg 1$ and the evolution behaviour of the total number is indeed controlled by migration processes of aggregates.

2.2. The case with $\bar{g}_1(t)/\bar{g}_2(t) \to \mathrm{const} > 0$ at $t \to \infty$

We then discuss the case in which $\bar{g}_1(t)/\bar{g}_2(t) \to C_1$ (C_1 is a constant greater than zero) at $t \gg 1$. In this case, both the self-birth and self-death processes cannot be ignored, and we find that the solution of $a_k(t)$ depends strongly on the value of C_1.

2.2.1. The $C_1 \geqslant 1$ subcase. We can conclude from equation (9) that the total mass increases with time for $C_1 > 1$ and remains at a constant value for $C_1 = 1$. Thus, the species can survive at the end. Moreover, in the $C_1 > 1$ subcase the total mass of the species will diverge if the average birth rate decreases less rapidly than t^{-1}.

On the other hand, from equation (14) one can determine the asymptotic solution of $a_k(t)$ at large times as follows:

$$a_k(t) \simeq A_0^{-1} t^{-2} \exp[(1 - C_1)t\bar{g}_2(t)][\bar{f}(t)]^{-2} \exp\left\{-\frac{k}{A_0 t \bar{f}(t) \exp[(C_1 - 1)t\bar{g}_2(t)]}\right\}. \tag{20}$$

Obviously, the aggregate size distribution also approaches the generalized scaling form of equation (18), and the typical aggregate size is $S(t) \simeq A_0 t \bar{f}(t) \exp[(C_1 - 1)t\bar{g}_2(t)]$. Moreover, we can also determine the total number of aggregates for this subcase. It is found that at large times, the asymptotic solution of the total number is the same as equation (19). It is not surprising because $M_1(t)f(t) \gg g_2(t)$ ($t \gg 1$) is also held for this subcase.

2.2.2. The $C_1 < 1$ subcase. In this subcase, the total mass of aggregates always decreases with time and the species survives finally only if the average death rate decays faster than t^{-1}. Moreover, from equation (14) we find that the aggregate size distribution is dependent strongly on the relation between $\exp[(1-C_1)t\bar{g}_2(t)]$ and $t\bar{f}(t)$ at large times.

When $\lim_{t\to\infty}\exp[(1-C_1)t\bar{g}_2(t)]/t\bar{f}(t)\to\infty$, we obtain the asymptotic solution of the aggregate size distribution,

$$a_k(t) \simeq \frac{A_0(1-C_1)^2\exp[(1-C_1)t\bar{g}_2(t)]}{\{\exp[(1-C_1)t\bar{g}_2(t)]+A_0(1-C_1)t\bar{f}(t)\}^2}C_1^k\left\{1+\frac{A_0(1-C_1)^2t\bar{f}(t)}{C_1\exp[(1-C_1)t\bar{g}_2(t)]}\right\}^{k-1}. \tag{21}$$

In the region of $t\gg 1$ and $k\gg 1$, equation (21) can be asymptotically rewritten as

$$a_k(t)\simeq A_0(1-C_1)^2\exp[(C_1-1)t\bar{g}_2(t)]C_1^k\exp\left[\frac{k}{S(t)}\right] \tag{22}$$

where

$$S(t)=\frac{C_1\exp[(1-C_1)t\bar{g}_2(t)]}{A_0(1-C_1)^2t\bar{f}(t)}. \tag{23}$$

Equation (22) indicates that the aggregate size distribution in this subcase does not satisfy the generalized scaling form, but it satisfies the modified scaling form [19, 20]

$$a_k(t)\simeq\lambda^k[\alpha(t)]^{-1}\Phi\left[\frac{k}{S(t)}\right]\qquad S(t)\propto\beta(t) \tag{24}$$

where λ is a constant ($0<\lambda<1$). The modified scaling form (24) indicates that there are two different scales associated with the aggregate size distribution. One is the growing scale $S(t)=A_0^{-1}C_1(1-C_1)^{-2}t^{-1}[\bar{f}(t)]^{-1}\exp[(1-C_1)t\bar{g}_2(t)]$, which is forced by the migration, birth and death processes. Another is a time-independent scale, $S=\lim_{t\to\infty}\sum_{k=1}^{\infty}k^2a_k(t)\big/\sum_{k=1}^{\infty}ka_k(t)\simeq(1+\lambda)/(1-\lambda)=(1+C_1)/(1-C_1)$, which will dominate the evolution behaviour of the aggregate size distribution in the long-time limit. Moreover, the scaling function in equation (24) is exponential, namely, $\Phi(x)=\exp(x)$.

In addition, we can determine the total number at large times,

$$M_0(t)\simeq A_0(1-C_1)\{\exp[(1-C_1)t\bar{g}_2(t)]+A_0(1-C_1)t\bar{f}(t)\}^{-1}. \tag{25}$$

Obviously, the total number decreases with time and tends to zero finally. Moreover, since $\exp[(C_1-1)t\bar{g}_2(t)]\to 0$ at $t\to\infty$, it follows from equation (22) that all aggregates in the system will vanish at the end.

When $\lim_{t\to\infty}\exp[(1-C_1)t\bar{g}_2(t)]/t\bar{f}(t)\to C_2$ (C_2 is a positive constant), the aggregate size distribution can be determined as follows:

$$a_k(t)\simeq A_0\frac{C_2(1-C_1)^2}{[C_2+A_0(1-C_1)]^2}[t\bar{f}(t)]^{-1}C_3^k\exp\left[\frac{k}{C_4t\bar{f}(t)}\right] \tag{26}$$

where $C_3=[C_1C_2+A_0(1-C_1)][C_2+A_0(1-C_1)]^{-1}$ and $C_4=[C_2+A_0(1-C_1)][C_1C_2+A_0(1-C_1)]C_2^{-1}(1-C_1)^{-2}$. Obviously, $C_3<1$. Thus, the aggregate size distribution in this subcase also approaches the modified scaling form (24) with the growing scale $S(t)\simeq C_4t\bar{f}(t)$ and the time-independent scale $S\simeq(1+C_3)/(1-C_3)$.

When $\lim_{t\to\infty}\exp[(1-C_1)t\bar{g}_2(t)]/t\bar{f}(t)\to 0$, from equation (14) we can deduce that for this subcase the aggregate size distribution also takes the form of equation (20).

Table 1. A summary of the scaling properties of $a_k(t)$ in different cases.

Case	Summary of the results
$\bar{g}_1(t)/\bar{g}_2(t) \to \infty$	(i) The aggregate size distribution satisfies the generalized scaling form (18).
$\bar{g}_1(t)/\bar{g}_2(t) \to C_1$ $(C_1 \geqslant 1)$	(i) The aggregate size distribution satisfies the generalized scaling form (18).
$\bar{g}_1(t)/\bar{g}_2(t) \to C_1(C_1 < 1)$	(i) If $\lim_{t\to\infty} \exp[(1-C_1)t\bar{g}_2(t)]/t\bar{f}(t)$ tends to a positive finite value, the aggregate size distribution approaches the modified scaling form (24); (ii) if $\lim_{t\to\infty} \exp[(1-C_1)t\bar{g}_2(t)]/t\bar{f}(t) \to 0$, the aggregate size distribution satisfies the generalized scaling form (18).
$\bar{g}_1(t)/\bar{g}_2(t) \to 0$	(i) If $\exp[t\bar{g}_2(t)]/\xi(t) \to \infty$, the aggregate size distribution takes a scale-free form; (ii) if $\exp[t\bar{g}_2(t)]/\xi(t) \to C_5$, the aggregate size distribution obeys the modified scaling form (24); (iii) if $\exp[t\bar{g}_2(t)]/\xi(t) \to 0$, the aggregate size distribution satisfies the generalized scaling form (18).

2.3. The case with $\bar{g}_1(t)/\bar{g}_2(t) \to 0$ at $t \to \infty$

We then discuss the case in which the average death rate is much larger than the corresponding average self-birth rate (namely, the self-death process dominates the system). The total mass in this case decreases exponentially as $\exp[-t\bar{g}_2(t)]$ and the species survives finally only if the average death rate $\bar{g}_2(t)$ decays faster than t^{-1}.

In this case, the analytic solution of $a_k(t)$ is dependent strongly on the value of $\lim_{t\to\infty} \exp[t\bar{g}_2(t)]/\xi(t)$ at $t \to \infty$, where $\xi(t) = \int_0^t g_1(t') \exp\{[\bar{g}_2(t') - \bar{g}_1(t')]t'\}\,dt' + A_0 t\bar{f}(t)$. We then analyse the scaling properties of the aggregate size distribution in several subcases as follows.

2.3.1. The subcase of $\lim_{t\to\infty} \exp[t\bar{g}_2(t)]/\xi(t) \to \infty$. In this subcase, we obtain the asymptotic solution of the aggregate size distribution at large times,

$$a_k(t) \simeq A_0[\xi(t)]^{-1}\{\xi(t)\exp[-t\bar{g}_2(t)]\}^k. \tag{27}$$

Equation (27) shows that the aggregate size distribution takes a scale-free form and decays exponentially in size k. Moreover, we obtain the total number as follows:

$$M_0(t) \simeq A_0 \exp[-t\bar{g}_2(t)]. \tag{28}$$

Obviously, both the total mass and the total number decay to zero with time, and thus all aggregates will vanish at the end.

2.3.2. The subcase of $\lim_{t\to\infty} \exp\,[t\bar{g}_2(t)]/\xi(t) \to$ const > 0. We then discuss the subcase in which $\lim_{t\to\infty} \exp[t\bar{g}_2(t)]/\xi(t) \to C_5$ (C_5 is a finite constant). We introduce

$$\xi(t) = \frac{\exp[t\bar{g}_2(t)]}{C_5} + \tau(t) \tag{29}$$

where $\tau(t)$ is a time-dependent function and tends to zero at large times. Substituting equation (29) into equation (14) and considering the condition $\bar{g}_1(t)/\bar{g}_2(t) \to 0$ at $t \gg 1$, we determine the asymptotic solution of $a_k(t)$ at large times,

$$a_k(t) \simeq A_0 C_5^2 (C_5+1)^{-k-2} \exp[-t\bar{g}_2(t)] \exp\left[\frac{k}{S(t)}\right] \tag{30}$$

with

$$S(t) \simeq \frac{(C_5+1)\exp[t\bar{g}_2(t)]}{C_5^2 \tau(t)}. \tag{31}$$

Equation (30) indicates that the aggregate size distribution approaches the modified scaling form of equation (24). Meanwhile, we determine the total number of aggregates,

$$M_0(t) \simeq A_0 \frac{C_5}{C_5 + 1} \exp[-t\bar{g}_2(t)]. \tag{32}$$

Moreover, since $\exp[-t\bar{g}_2(t)] \to 0$ at $t \gg 0$, the aggregate size distribution consistently decays with time and decreases to zero finally. Hence, all aggregates will vanish at the end.

2.3.3. The subcase of $\lim_{t\to\infty} \exp[t\bar{g}_2(t)]/\xi(t) \to 0$. In this subcase, one can derive the scaling solution of the aggregate size distribution,

$$a_k(t) \simeq A_0 \exp[t\bar{g}_2(t)][\xi(t)]^{-2} \exp\left[-\frac{k}{S(t)}\right] \tag{33}$$

with the typical size

$$S(t) \simeq \xi(t) \exp[-t\bar{g}_2(t)]. \tag{34}$$

The result shows that for this subcase, the aggregate size distribution obeys the generalized scaling form (18). Furthermore, we determined the total number of the aggregates in the long-time limit,

$$M_0(t) \simeq A_0[\xi(t)]^{-1}. \tag{35}$$

Obviously, when time is large, the total number of the aggregates will tend to zero. Moreover, equation (33) indicates that $a_k(t)$ decays with time and vanishes at the end. Thus, all aggregates cannot survive finally.

3. Summary

In this work, we have studied a solvable kinetic model of aggregate growth, in which monomers can spontaneously migrate from one aggregate to another; meanwhile, monomer birth and death processes may occur in any aggregates. Considering the rates of migration, self-birth and self-death processes may be time dependent, we assume the migration rate kernel $I(i, j; t) = ijf(t)$, the self-birth kernel $J_1(i; t) = ig_1(t)$ and the self-death rate kernel $J_2(i; t) = ig_2(t)$. Based on the mean-field rate equation, we investigated the analytic solution of the aggregate size distribution and then analysed the kinetic scaling properties of the system.

Consider a relatively realistic case (e.g., the evolution of city population), in which $f(t)$ is an increasing function, and $g_1(t)$ and $g_2(t)$ are both decreasing functions. The results showed that the kinetic behaviour of the aggregate size distribution $a_k(t)$ depends crucially on the relation between $g_1(t)$ and $g_2(t)$. Moreover, the migration rate kernel also plays an important role in the evolution of $a_k(t)$. In the long-time limit, the aggregate size distribution can take the generalized or modified scaling form in some cases while it has a scale-free form in other cases, which is illustrated in table 1. When $f(t)$, $g_1(t)$ and $g_2(t)$ all tend to finite constants at large times, our results can asymptotically reduce to those obtained in the same model but with time-independent rate kernels [15, 16]. More intriguingly, in the case of birth rate less than death rate, the aggregate size distribution in the time-dependent-rate-kernel system can satisfy the generalized or modified scaling form (see table 1), while it always satisfies the modified scaling form in the system with time-independent rate kernels [16]. This indicates that besides the size dependence of the rate kernels, the time dependence also plays an important role in the scaling properties of the aggregate size distribution.

Moreover, the total mass in the system also has abundant evolution behaviours. When $\bar{g}_1(t)/\bar{g}_2(t)$ tends to infinity or a constant greater than 1 at large times, the total mass increases with time and the aggregates can survive. When $\bar{g}_1(t)/\bar{g}_2(t)$ is strictly equal to 1, the total mass of the system is formally conserved and remains at the initial value A_0. When $\bar{g}_1(t)/\bar{g}_2(t)$ tends to a constant less than 1, the total mass decreases with time. Additionally, the total number always decreases with time for all cases.

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